

# How to reach the orbital configuration of the inner three planets in HD 40307 Planet System ?

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## ABSTRACT

The formation of the present configuration of three hot super-Earths in the planet system HD 40307 is a challenge to dynamical astronomers. With the two successive period ratios both near and slightly larger than 2, the system may have evolved from pairwise 2:1 mean motion resonances (MMRs). In this paper, we investigate the evolutions of the period ratios of the three planets after the primordial gas disk was depleted. Three routines are found to probably result in the current configuration under tidal dissipation with the center star, they are: (i) through apsidal alignment only; (ii) out of pairwise 2:1 MMRs, then through apsidal alignment; (iii) out of the 4:2:1 Laplace Resonance (LR), then through apsidal alignment. All the three scenarios require the initial eccentricities of planets  $\sim 0.15$ , which implies a planetary scattering history during and after the gas disk was depleted. All the three routines will go through the apsidal alignment phase, and enter a state with near-zero eccentricities finally. We also find some special characteristics for each routine. If the system went through pairwise 2:1 MMRs at the beginning, the MMR of the outer two planets would be broken first to reach the current state. As for routine (iii), the planets would be out of the Laplace Resonance at the place where some high-order resonances are located. At the high-order resonances 17:8 or 32:15 of the planets c and d, the system will possibly enter the current state as the final equilibrium.

*Subject headings:* planets and satellites: formation - planets and satellites: HD40307

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## 1. Introduction

Searching for Earth-size planets is one of the most exciting objectives for the present exoplanet hunting. According to the estimates from the results of the Kepler mission, more than 30% of the stars will host planets with mass less than 10 Earth masses (so called super Earths), and the percent is even larger for the planets around small-mass stars like M dwarf (Howard et al. 2012; Dressing & Charbonneau 2013). Based on the core accretion planet formation theory, super Earths were originally formed in distant orbits, and migrate inward under the interaction with the gas disk (Ward 1997; Tanaka et al. 2002; Zhou et al. 2005). Researchers found through numerical simulations that planets are very likely to enter mean motion resonances (MMRs) during the convergent migration processes (Terquem et al. 2007). Some exo- planet pairs are observed to be in exact MMRs, e.g., GJ 876b,c,d (Rivera et al. 2010). However, the statistics from Kepler planet candidates shows that most of the planet pairs are in near MMRs with period ratios slightly larger than the exact integer ratios (Fabrycky et al. 2012; Batygin & Morbidelli 2013).

Several mechanisms have been proposed to explain the origin of those period ratios slightly larger than the integer ratios. For short period planets, star-planet tidal interactions would deplete the semi-major axis of the inner planet more quickly and make the period ratio dispersing (Lithwick & Wu 2012; Batygin & Morbidelli 2013). For long period planets, Baruteau & Papaloizou (2013) proposed that the gravitational interactions between partial gap-opening planets and the gas disk may instead provide efficient dissipation. Besides, Lee et al. (2013) raised that tides between planets and star are not strong enough to increase part of the planet pairs to the current separations, and it is also uncertain that the migration of planets in the disk could result in the observed distribution of planetary period ratios because of the complex disk environment. Wang et al. (2012) worked on the formation of a near Laplacian resonance configuration in the KOI-152 system, and focused on the influence of stellar accretion, stellar magnetic field and the speed of migration in the protoplanetary disk.

HD 40307 system plays a notable role and its migration history has been studied in detail representatively. The three inner planets in this system have the period ratios 2.23 and 2.13 for the inner and outer pairs, respectively (Mayor et al. 2009). Both Zhou (2010) and Papaloizou & Terquem (2010) (PT2010 hereafter) simulated their inward migration in a gas disk, and found that the three planets were trapped into pairwise 2:1 MMR or Laplace Resonance (LR) or both of them, and retained in it until the tidal dissipation between star and planets is effective, which may drive it out of the resonances. However, how the exact final configuration is achieved under different initial conditions (pairwise 2:1 MMRs or LR, etc.) is not fully understood. PT2010 also demonstrated that in low-eccentricity situation,

the period ratios evolve too slowly to reach the current orbital architecture. Meanwhile, they claimed that the current state of the system cannot be originated from LR directly.

Recently, Tuomi et al. (2012) reanalyzed the RV data of the system HD40307, and claimed that it exists three additional planets (e,f,g) with masses of  $3.5m_E$ ,  $5.2m_E$  and  $7.1m_E$  at orbits with periods of  $34.62d$ ,  $51.76d$  and  $197.8d$ , respectively. If the presence of additional planets is confirmed, the planet system is very compact especially for the inner 5 ones. To reveal whether the additional outer planet might interact with the inner ones, we calculate the relative space among planets, and find that there are respectively 21, 17, 13, 11, 33 times of their mutual Hill’s radii among the 5 neighboring pairs. So dynamically the planets d,e,f are more closely related. Assuming they have around 10 Earth masses, the orbital crossing time could be around  $10^{8-9}$  times of their periods, i.e.,  $10^{7-8}$  years (See Eq. (3) or Figure 3 of Zhou et al. (2007)). This time scale is comparable to the above tidal evolution for  $Q' = 100$ . So if the outer three planets are confirmed, the inner five planets could evolve as a dynamically related system. However, since our discussed configurations are from MMRs, and only the inner three are effected significantly by tidal dissipations, the above three routines might not change too much unless the outer planets are also involved in MMRs. Meanwhile, Figure 1 gives the comparison of the eccentricities of the planets HD 40307 b, c, d. We can see the secular oscillations of the eccentricities are in the same order of magnitude whether the outer three planets are considered or not. According to these, the model we apply includes the inner three planets merely, which reduces the number of degree of freedom and simplifies the problem dramatically.

In this paper, we reconsider the process of tidal dissipation and study the final configurations of the inner three planets under different evolution scenarios. Three types of evolution routines have been explored to get to the observed state finally. We deduce that the apsidal alignment phase is a very common and maybe a necessary phase for such compact systems under tidal dissipation. In Section 2, we describe the numerical model. Section 3 discusses the possible current configuration of the three planets. Section 4 describes the three kinds of evolution paths which can get to the current configuration. Then we compare the three evolution paths and make some speculations in Section 5. The last section gives a summary of this paper.

## 2. Model

We consider a host star and N planets in a barycentric inertial coordinate system (N=3 for HD 40307 system). There are three additional forces besides gravitational interactions.

The equations of motion are:

$$\frac{d^2 \mathbf{r}_i}{dt^2} = \sum_{j=1, j \neq i}^n \frac{Gm_j(\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} + \mathbf{f}_{TD} + \mathbf{f}_{QD} + \mathbf{f}_{GR} \quad i = 0, \dots, 3, \quad (1)$$

where  $i=0$  represents the star, and  $i=1,2,3$  represents the planets b,c,d respectively.  $\mathbf{f}_{TD}$ ,  $\mathbf{f}_{QD}$  and  $\mathbf{f}_{GR}$  denote the acceleration produced by tidal damping, quadrupole moment and general relativistic respectively (Mardling & Lin (2002)). The specific expressions are

$$\begin{aligned} \mathbf{f}_{TD} = & -\frac{9n}{2Q'_p} \left(\frac{m_0}{m_p}\right) \left(\frac{S_p}{a}\right)^5 \left(\frac{a}{r}\right)^8 \\ & \cdot [3(\hat{\mathbf{r}} \cdot \dot{\mathbf{r}})\hat{\mathbf{r}} + (\hat{\mathbf{r}} \times \dot{\mathbf{r}} - r\boldsymbol{\Omega}_p) \times \hat{\mathbf{r}}] \end{aligned} \quad (2)$$

$$\begin{aligned} \mathbf{f}_{QD} = & \frac{S_p^5(1 + m_0/m_p)k_p}{r^4} \left\{ \left[ 5(\boldsymbol{\Omega}_p \cdot \hat{\mathbf{r}})^2 - \Omega_p^2 - \frac{12Gm_0}{r^3} \right] \right. \\ & \left. \cdot \hat{\mathbf{r}} - 2(\boldsymbol{\Omega}_p \cdot \hat{\mathbf{r}})\boldsymbol{\Omega}_p \right\} \end{aligned} \quad (3)$$

$$\begin{aligned} \mathbf{f}_{GR} = & -\frac{Gm_{0p}}{r^2 c^2} \left\{ \left[ (1 + 3\eta)\dot{\mathbf{r}} \cdot \dot{\mathbf{r}} - 2(2 + \eta)\frac{Gm_{0p}}{r} \right. \right. \\ & \left. \left. - \frac{3}{2}\eta\dot{\mathbf{r}}^2 \right] \hat{\mathbf{r}} - 2(2 - \eta)\dot{\mathbf{r}}\dot{\mathbf{r}} \right\} \end{aligned} \quad (4)$$

where  $m_p, S_p, \boldsymbol{\Omega}_p$  is the mass, radius, and spin speed of planet, respectively.  $m_{0p} = m_0 + m_p$ ,  $\eta = m_0 m_p / m_{0p}^2$ .  $\mathbf{r}, \dot{\mathbf{r}}$  is the position vector and speed vector of planet relative to the central star.  $c$  is the speed of light.  $Q'_p = 3Q_p / (2k_p)$ .

As the tidal dissipation from the planets deformation is much bigger than that from the star's, we consider the planetary tide merely. The minimum masses of all planets in this system have the same order of magnitude with the Earth, so we suppose the same damping parameter  $Q_p = 0.01$  and apsidal motion constant  $k_p = 0.3$  (Zhou et al. 2008; Batygin et al. 2009) for all three planets in all simulations. Terrestrial planets have  $Q_p = 10 - 100$  (Goldreich & Soter 1966) and we set a smaller value to accelerate the damping and shorten the calculating time as (Mardling 2007) and PT2010 did. Besides, we get the radii of planets by supposing the densities of the planets equal to Earth's. In fact, the radii are coupled with the tidal dissipation parameter  $Q'_p$  in the expressions of all additional forces, and  $Q'_p$  is inversely proportional to tidal damping timescale, so some deviations of radii or  $Q'_p$  are equivalent to a change of evolution time in most cases.

The timescale of planetary rotation during tidal damping is much shorter than that of orbital evolution, so we set  $\Omega_p = n$  and the spin axis is perpendicular to the orbital plane at the beginning. Subsequent evolution of  $\boldsymbol{\Omega}_p$  is given by the relation as follow (Mardling & Lin 2002)

$$\dot{\boldsymbol{\Omega}}_p = -\frac{m_0 m_p}{I_p(m_0 + m_p)} \mathbf{r} \times (\mathbf{f}_{TD} + \mathbf{f}_{QD}), \quad (5)$$

which is deduced by the conservation of total angular momentum, and  $I_p$  is inertial moment of planet.

We apply the RKF78 variable-step integrator to make the N-body simulations, and the additional forces are added during every step. The numerical error for every step is set to be  $10^{-12}$ , and the total energy is generally conserved to  $10^{-6}$  in the conservative cases (Ji et al. 2005). The integration time is about 10-12 hours for every run. The elements of planets are output in equal interval (every 100yrs) to track the evolutions.

### 3. The state of the three planets HD40307 b, c, d in low-eccentricity situation

We first set small eccentricities, and make the simulations with the observed semi-major axes and minimum masses (Table 3) (We adopt the minimum masses of the planets as fiducial values in our simulations, and the effects of more massive planets will be discussed in section 6), and altering initial eccentricities and phase angles. The resonance angles are given by  $\Phi_1 = 2\lambda_2 - \lambda_1 - \varpi_1$ ,  $\Phi_2 = 2\lambda_2 - \lambda_1 - \varpi_2$ ,  $\Phi_3 = 2\lambda_3 - \lambda_2 - \varpi_2$ ,  $\Phi_4 = 2\lambda_3 - \lambda_2 - \varpi_3$  (Lee & Peale 2002; Ji & Li 2002). Here  $\lambda_i$  and  $\varpi_i$  represent the mean longitude and the longitude of pericentre of planet  $i$ , and the indices  $i = 1, 2, 3$  stand for the planet b, c, d, respectively.

Then we find that as long as the eccentricities are small ( $\sim 10^{-4}$ ), whatever initial phase angles are set to be, the system would eventually come to the same equilibrium state, with  $\Phi_1, \Phi_3$  librating around 0, and  $\Phi_4$  “nearly librating” around  $\pi$  (“nearly librating” here means that the resonant angle has no obvious libration but is just more dense around some place and has long-term time average, or is associated with long term changes of the orbital elements). Figure 2 shows the equilibrium state in the  $e - \Phi$  phase space. And it is consistent with the simulations in PT2010.

Delisle et al. (2012) gave a global analysis of the phase space of the situation above, and demonstrated that the apparent libration of the resonant angles in low eccentricities situation results from the severe damping of the amplitudes of the eigenmodes in the secular motion, and the planets are not really in the MMRs. Indeed, under tidal dissipation, the separatrices that exist in the resonant systems eventually disappear when the eccentricities of planets are very small. There is only a circulation of the orbits around a single elliptical fixed point left in the phase space (see Fig. 2 in Delisle et al. (2012)).

#### 4. Paths that will evolve to the present configurations

After the gas disk disappears, tidal dissipation between the star and planets will dominate the migration of planets in close-in orbits, which basically cause inward migration when the planets are inside the synchronous orbit of stellar spin. The decayed timescale of the semi-major axis due to planetary tides can be estimated as  $\tau_a = \tau_{\text{circ}}/e^2$  (Lithwick & Wu 2012; Zhou et al. 2008), where  $\tau_{\text{circ}}$  is the orbital circularization time scale due to tide, and  $e$  is the orbital eccentricity of the planet. So different eccentricities and the relative magnitudes of three planets' eccentricities would correspond to different evolution directions of the period ratios of the adjacent planets. And the orbital eccentricities are mainly determined by the orbital configurations and the resonance types.

In this section we focus on the moderate-eccentricity situation and investigate different configurations and resonances that the planets may have gone through. As collisions or scattering are very likely to take place during the gas disk dissipation for such a compact system, high eccentricities would be common before tidal damping effects (Ogihara et al. 2010). PT2010 pointed out that if the eccentricity of the outermost planet is up to 0.15, the period ratios would shift to the nearby current values from the values close to 2. Our simulations are generally consistent with PT2010. Further more, we make comparisons and classifications, and find three kinds of paths which can get to the current state from different initial states.

##### *Path 1: Apsidal alignment.*

Assume that the three planets were formed around the moderate region ( $\gtrsim 1$  AU) one after another, and the first-formed planet migrated inward first, and then the three planets may have a history that were far away from any MMRs. After the gas disk disappears, they would undergo secular evolution under their mutual interactions. To investigate this type of evolution with emphases on the final configuration, we fix the initial conditions  $P_2/P_1 = 2.055$ ,  $P_3/P_2 = 2.12$ ,  $e_1 = e_2 = e_3 = 0.1$ , so that the orbits can evolve to around the present configuration. The evolution is shown in Figure 3.

One of the major characteristics for this evolution under tidal dissipation is that the planets will be quickly driven into three-body secular phase locking, or apsidal alignment ( $\varpi_1 \approx \varpi_2 \approx \varpi_3$ ). The reason is that, orbital alignment (i.e.,  $\Delta\varpi = 0$ ) is a quasi-equilibrium state in the  $e - \Delta\varpi$  plane (see Fig. 1 of Mardling (2007), also Zhou & Sun (2003) for the non-dissipation cases). Though the planets in our cases are too close to be approximated by a hierarchical system, the evolution scenario here is quite similar to that in a hierarchical system, as shown in Mardling (2007). First, the three planets align quickly. During the alignment process, the amplitudes of the oscillations of eccentricities decrease to  $\sim 10^{-4}$

within  $3 \times 10^5$  years for  $Q_1 = 0.01$ , which corresponds to 3 Gyrs provided  $Q' = 100$ . The alignment configuration is kept until the eccentricities are damped to almost 0. Then the alignments of apsidal lines are broken, and the system turns to the low-eccentricity state (the same as shown in Figure 2).

Another feature of the evolution path is that  $P_3/P_2$ , the period ratio of the outer pair, has no significant change in this process. Thus the evolution track in the  $(P_2/P_1, P_3/P_2)$  plane is almost a line parallel to the x-axis until the end of evolution. Figure 4 emphasizes the feature further. It shows the evolution tracks of 13 orbits with different initial  $P_1$  according to  $P_2/P_1 = 2.01, 2.02, 2.03, \dots, 2.13$ . Why does  $P_3/P_2$  change slightly in these evolution paths? On the one hand, orbital angular momentum and energy transfers among different planets are much weaker than that in any two-body mean motion resonance or three-body resonance. On the other hand, the tidal dissipation of the middle and outermost planets is not obvious as the planets are not close enough to the star. Accordingly, we infer that the period ratio  $P_3/P_2$  should have approached the current value before the system began the apsidal-alignment evolution. So this path could just be as an intermediate stage if the system was in 2:1 MMRs before tidal evolution.

*Path 2: pairwise 2:1 MMRs.*

Migration of planets embedded in the protoplanetary disk is very common (Goldreich & Tremaine 1979; Ward 1986, 1997; Tanaka et al. 2002). Zhou (2010) and PT2010 specifically simulated the three planets in HD40307 system migrating in the gas disk, and both found that the planets are easily trapped into pairwise 2:1 MMRs or Laplace Resonance during the migration. Assuming the initial configurations  $P_2/P_1 \approx 2, P_3/P_2 \approx 2$ , we investigate the subsequent evolution of three planets under tidal effects with the star.

Figure 5 shows two typical orbits with different initial eccentricity  $e_1$ , and one of them reaches the observed state finally. The resonances are very unstable and are disrupted  $10^3 - 10^4$  years later with  $Q'=0.01$  (corresponds to  $10^7 - 10^8$  years for  $Q = 100$ ). The two examples show mainly two types of breakup of MMRs: in the left case, the outer 2:1 MMR goes out first, and then the middle planet continues to be dragged in by the innermost planet because of the inner 2:1 MMR, which makes  $P_3/P_2$  increasing quickly to the present value. Instead, if the inner pair of planets go out of the 2:1 MMR at first, like the case in the right, then  $P_3/P_2$  would keep around 2, because without the inner 2:1 MMR, the middle planet could not move inward more than the outermost planet. So for this kind of path, the outer pair being out of MMR first is the necessary precondition for the system coming to the present configuration.

*Path 3: Laplace Resonances.*

Two successive period ratios both approaching 2 also remind us whether the three planets are in LR. The LR is defined as  $n_1 - 3n_2 + 2n_3 \approx 0$  so that  $\Phi_L = \lambda_1 - 3\lambda_2 + 2\lambda_3$  liberates around either  $0^\circ$  or  $180^\circ$ . Such a configuration has been discovered and investigated among the Galilean satellites of Jupiter (Peale & Lee 2002). The satellites go through either the primordial inward migration due to interactions with a circumjovian disk, or subsequent differential orbital expansions from tides raised on Jupiter, and then were trapped into pairwise 2:1 MMRs as well as LR with  $\Phi_L$  liberating around  $\pi$ .

PT2010 has clarified a negative conclusion based on the planets' mean motions not satisfying the Laplace relation  $3n_2 - 2n_3 - n_1 = 0$ . In spite of that, we find LR still a possible part of evolution process, and such a case is given in Figure 6. Two cases in the figure have the same initial conditions except for phase angles. The case in dash line enters LR at first, and the period ratios go along the Laplace relation at this duration (the dot black line). Then LR is broken at some place, and secular evolution follows before a tidal equilibrium comes. Compared to this, with the different initial phase angles, the case in solid starts secular evolution directly, and the final equilibrium is far away from the current. In the case, the periods of the outer two planets hardly change during the evolution, while they vary a lot in the LR case due to the strong interaction among planets. Hence, LR trapping becomes the key step to shift  $P_3/P_2$ , and propel the system into the current under this kind of initial conditions.

Trapping into LR is a quite stochastic event, and mainly depends on the phase angles of planets at the moment when the Laplace relation is satisfied. However, there seems to be some rules to follow the place where the system is out of LR. We made a scan on the oscillating amplitude of the Laplace angle for different period ratios and different eccentricities, by N-body simulation without dissipation (Figure 7). We found that for the same eccentricities, there are some places where LR is more unstable, such as  $P_3/P_2 \simeq 2.125, 2.143, \dots$ , which corresponds to the high-order resonances  $17/8, 15/7, \dots$  of the outer two planets. Under tidal dissipation,  $P_3/P_2$  increases and eccentricities decrease, so the corresponding position of the state in the  $P_3/P_2 - e$  phase space will move toward the lower-right and encounter a series of the high-order resonances. For the cases approaching the current state finally,  $17/8$  or  $32/15$  would be the high-order resonance from which the system comes out of the LR. Because after out of the LR, the system will enter the apsidal-alignment state, in which the outer period ratio  $P_3/P_2$  will not change a lot.



## 5. Comparison and some speculations

We compare the states the system goes through in the three kinds of paths, and find that although the planets stem from different states, they all include the processes of apsidal alignment and the following low-eccentricity equilibrium (Figure 8). The final equilibrium state has no difference in the three kinds of paths, so we still cannot tell the exact story even though the eccentricities or the resonance angles have been detected precisely. However, due to the secular evolution as a common state in these evolution histories (the horizontal part of paths in Figure 8 left), if the current eccentricities are  $\sim 10^{-4}$  as figure 2 shows, it would imply that the supposed robust events during which the eccentricities were excited must have occurred at least  $\sim 10^5 Q'/0.01yr$  ago, the time of the secular evolution in this system.

## 6. Conclusion and discussion

We have investigated the possible evolution histories of the inner three planets in HD40307 system. We use the N-body model, adding tides raised by the star on the planet and the general relativity as the additional forces. We find three kinds of paths along which the system can evolve to the current configuration. The three paths all need moderate eccentricities ( $\sim 0.15$ ), which are supposed to result from some robust events, such as collision or scattering. Moreover, the three paths originate from different areas in the  $P_2/P_1$  versus  $P_3/P_2$  phase space, while they all pass the apsidal alignment duration before the final tidal equilibrium arrives.

Minimum masses are used in all cases above. We also made some cases with twice minimum masses, and found that the stronger effects among planets cause higher eccentricities. However, the actual evolution time is proportional to the damping parameter  $Q'$ , and Neptune-like planets tend to have bigger  $Q'$  than Earth-like planets. As a result, the evolutions for more massive planets should not be faster than the ones for minimum masses. Another assumption we have made is that the three planets'  $Q'$  are the same or at least in the same magnitude, and it should be most likely to be the truth due to their minimum masses in the same magnitude. But in case this is not true, which means these planets might have totally different components, then the evolution process would be different from what we have discussed. All these are waiting for a further detection.

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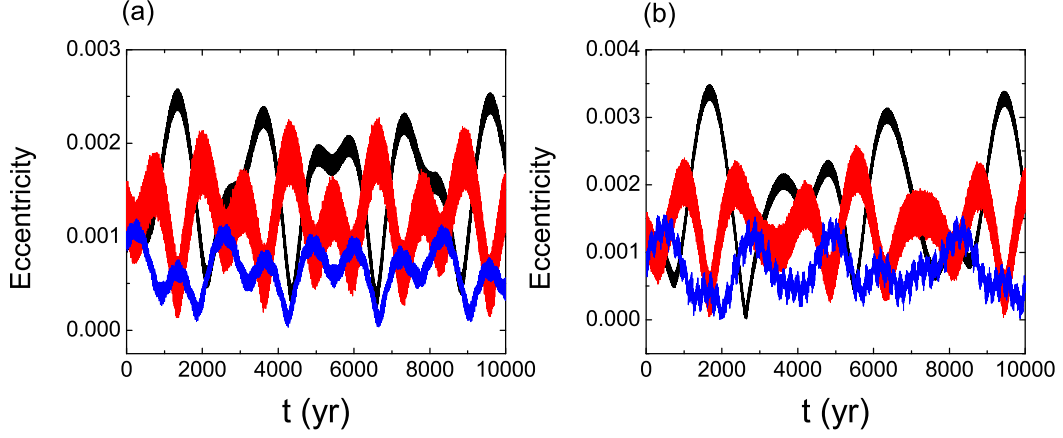


Fig. 1.— The evolution of the eccentricities of the planets HD40307 b,c,d, without (a) and with (b) the outer planets HD40307 e,f,g.

Table 1: Orbital elements of HD40307 b,c,d and the star from Mayor et al. (2009).

Parameter		HD40307 b	HD40307 c	HD40307 d
$m_2 \sin i$	$[M_\oplus]$	4.2	6.9	9.2
$P$	[days]	$4.3115 \pm 0.0006$	$9.620 \pm 0.002$	$20.46 \pm 0.01$
$a$	[au]	0.047	0.081	0.134
$e$		0.0	0.0	0.0
Star	Mass	Sp. type	Metallicity	$T_{eff}$
HD40307	$[M_\odot]$		[dex]	[K]
	$0.77 \pm 0.05$	K2.5V	$-0.31 \pm 0.03$	$4977 \pm 59$

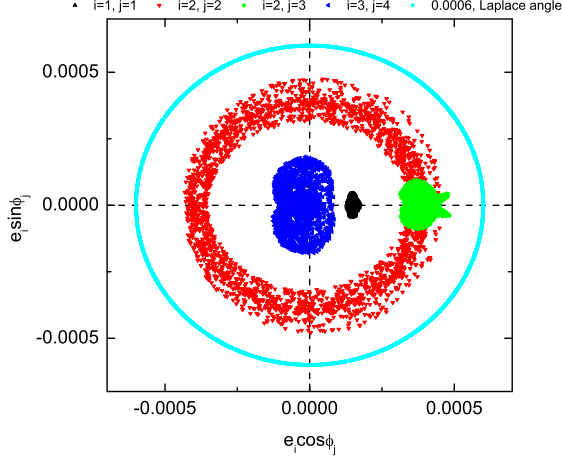


Fig. 2.— The equilibrium state of HD40307 system under tidal dissipation. It shows that  $\Phi_1$ (black dots),  $\Phi_3$ (green dots) librate around 0, and  $\Phi_4$ (blue dots) nearly librates around  $\pi$ . The eccentricities in the equilibrium are in the order of magnitude of  $10^{-4}$ . Laplace angle(cyan dots) is circulating.

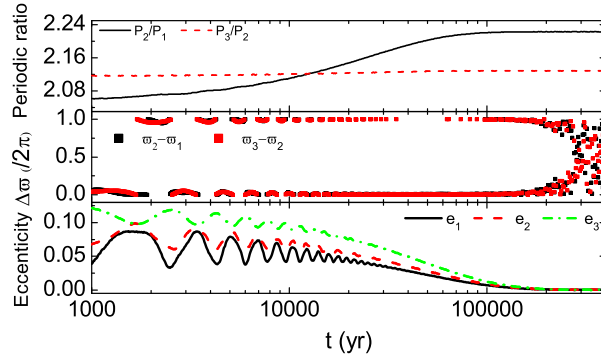


Fig. 3.— Period ratio (top), differences of longitude of pericentre (middle) and eccentricities (bottom) evolve with time in one simulated run. Three planets are initially located at 0.05 au, 0.08 au and 0.134 au ( $P_2/P_1 = 2.055$ ,  $P_3/P_2 = 2.12$ ), with the same eccentricities 0.1. The phase angles are chosen arbitrarily. The apsidal alignments ( $\Delta\varpi \approx 0$ ) are kept until the eccentricities are damped to very small values.

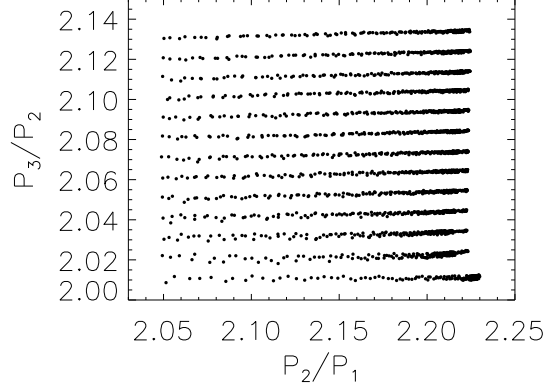


Fig. 4.— Evolution tracks of 13 orbits with  $P_2/P_1 = 2.01, 2.02, \dots, 2.13$  are plotted in the  $P_2/P_1 - P_3/P_2$  plane. Other initial conditions are  $P_3 = 20.5$  days,  $P_3/P_2 = 2.05$ ,  $e_1 = 0.15$ ,  $e_2 = e_3 = 0.0001$ . Angular parameters are set randomly.

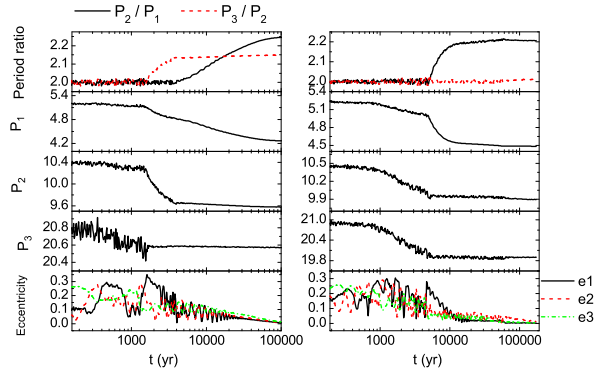


Fig. 5.— Two examples for three planets initially in the pairwise 2:1 mean motion resonances with high eccentricities. Each column shows one example. In the left, three planets are put at 0.054, 0.085 and 0.136 au respectively ( $P_2/P_1 = P_3/P_2 = 2.0001$ ), with  $e_1 = 0.07$ ,  $e_2 = 0.25$ ,  $e_3 = 0.19$ . In the right, only the inner planet's initial eccentricity is different,  $e_1 = 0.19$ . In the left case, the outer pair of planets are first out of 2:1 MMR, which causes the two period ratios reaching up to around the present position finally.

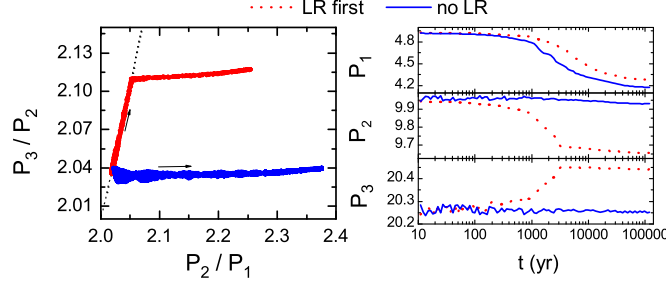


Fig. 6.— The comparison of two cases originally in the Laplace relation. The two cases have the same parameter,  $P_2/P_1 = 2.02$ ,  $P_3/P_2 = 2.04$ ,  $P_3 = 20.2865\text{days}$ ,  $e_1 = 0.19$ ,  $e_2 = 0.19$ ,  $e_3 = 0.01$ , with different phase angles. The left panel is the period ratio tracks, and the right ones are the period evolutions of three planets versus time.

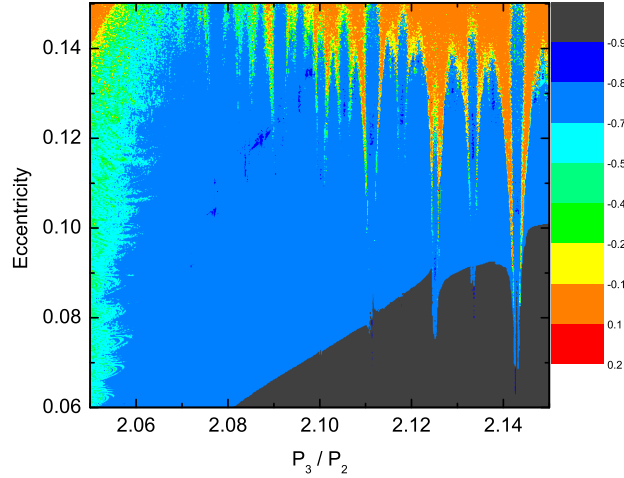


Fig. 7.— Contour of the oscillating amplitudes of the Laplace angle. Blue end of the color bar represents libration, and red end means circulation. Every case is integrated  $10^5$  yrs without tidal dissipation. As for the initial conditions, the outermost planet is fixed at 20.8 days,  $P_2/P_1$  is calculated using  $P_3/P_2$  and the Laplace relation. Three planets have the same initial eccentricity for reducing the variations. Initial phase angles are set as  $\varpi_1 = 0$ ,  $\varpi_2 = \pi$ ,  $\varpi_3 = 0$ ,  $\lambda_1 = 0$ ,  $\lambda_2 = \pi$ ,  $\lambda_3 = 0$ , which can make the system enter LR easily. From the panel, we can see some more unstable place of LR on  $P_3/P_2 \simeq 17/8, 32/15, 15/7$ . These places should be the reason why Laplace Resonance is broken in the dash case of Figure 6.

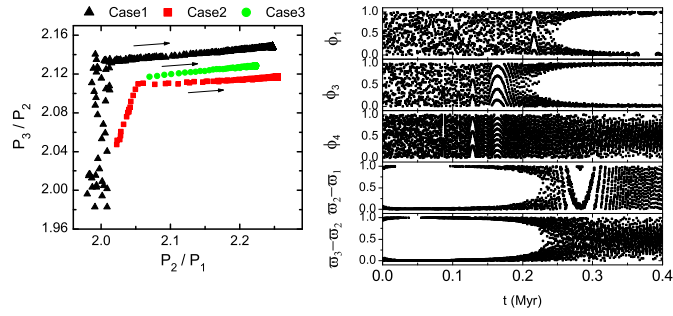


Fig. 8.— The left panel shows the paths of  $P_2/P_1$  versus  $P_3/P_2$  of three representative cases, which are originated from different initial conditions, and get to around the current state of HD40307 finally. All of the paths have a parallel part, which corresponds to a secular evolution with apsidal alignment. Moreover, the runs turn into the same state at the end, which is also the one in Figure 2. The right three panels show the resonance angles and differences of longitude of pericentre of Case 3 versus time. For the other two cases, these phase angles evolve similarly when they enter the apsidal alignment part, and are not shown here.